MATH3280A Introductory Probability, 2014-2015
Solutions to HW3

P.136 Ex.4

Solution
Let E_1, E_2, and E_3 be the events that a person has disease D_1, D_2 and D_3 respectively.
Let F be the event that a person has symptom A.

The assumption that no one carries more than one of these three diseases means that E_1, E_2, and E_3 are mutually disjoint.
The assumption that the only possible causes of symptom A are D_1, D_2 and D_3 means that $F \subset E_1 \cup E_2 \cup E_3$.

Then we have $F = FE_1 \cup FE_2 \cup FE_3$, which is a disjoint union.

$$\begin{aligned} P(F) &= P(FE_1 \cup FE_2 \cup FE_3) \\ &= P(FE_1) + P(FE_2) + P(FE_3) \\ &= P(F|E_1)P(E_1) + P(F|E_2)P(E_2) + P(F|E_3)P(E_3) \\ &= 0.5 \times 0.05 + 0.7 \times 0.02 + 0.8 \times 0.035 \\ &= 0.067 \end{aligned}$$

Hence 6.7% of the population have symptom A.

□

P.136 Ex.7

Solution
Let E_A, E_B and E_C be the events that a prisoner who tried to escape used road A, B and C respectively.
Let S be the event that a prisoner succeeded in escaping.

By Bayes' Formula, we have

$$P(E_C|S) = \frac{P(S|E_C)P(E_C)}{P(S|E_A)P(E_A) + P(S|E_B)P(E_B) + P(S|E_C)P(E_C)}$$

$$= \frac{0.08 \times 0.2}{0.2 \times 0.3 + 0.25 \times 0.5 + 0.08 \times 0.2}$$

$$= \frac{16}{201} \approx 0.0796$$

Hence the probability that a prisoner who succeeded in escaping used road C is $\frac{16}{201}$.

□